

Correlation induced memory effects in the transport properties of low dimensional systems

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We demonstrate the remnant presence of *initial* correlations in the *steady-state* electrical current flowing between low-dimensional interacting leads. The leads are described as Luttinger liquids and electrons can tunnel via a quantum point-contact. We derive an analytic result for the time-dependent current and show that ground-state correlations have a large impact on the relaxation and long-time behavior. In particular, the I-V characteristic cannot be reproduced by quenching the interaction in time. We further present a universal formula of the steady-state current j_S for an arbitrary sequence of interaction quenches. It is established that j_S is history dependent provided that the switching process is non-smooth.

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Non-equilibrium phenomena in open nanoscale systems offer a formidable challenge to modern science[1]. Controlling the electron dynamics of a molecular device is the ultimate goal of nanoelectronics and quantum computation[2]; its microscopic description a problem at the forefront of statistical quantum physics[3]. Resorting to approximate methods is inevitable to progress.

Standard many-body techniques consider an initial state with *no interaction* and with *no contact* between the system and the baths (leads from now on), and then switch them on in time[4–6]. In fact, it is plausible to believe that starting from the true *interacting* and *contacted* state the long-time results would not change. To what extent, however, such belief is actually the truth? This question is of both practical and fundamental interest. It has been shown by us [7] and others [8] that for non-interacting electrons the initial contact plays no role at the steady-state [9] (theorem of equivalence). Allowing for interactions in the system only (non-interacting leads) Myöhänen *et al.* found that steady-state quantities are not sensitive to initial correlations either[10]. It is the purpose of this Letter to show that interacting leads change dramatically the picture: the switching process can indeed have a large impact on the *relaxation* and the *steady-state* behavior.

We consider two one-dimensional interacting leads described as Luttinger Liquids (LL)[11], see Fig. 1. It is known that a LL does not relax to the ground state after a sudden quench of the interaction [12–14] (thermalization breakdown). The implications of such important result in the context of time-dependent (TD) transport are totally unknown and will be here explored for the first time. We compare the dynamics of initially (a) contacted ($\eta_T = 1$) versus uncontacted ($\eta_T = 0$) and (b) interacting ($\eta_I = 1$) versus non-interacting ($\eta_I = 0$) LL when driven out of equilibrium by an external bias. Our main findings are

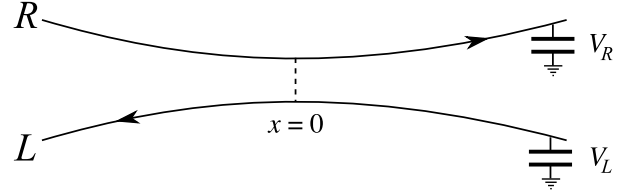


FIG. 1. Sketch of the device. Two interacting leads hosting L and R movers are connected at $x = 0$ via a weak link. A bias voltage $V_L - V_R$ can be applied between the leads.

that in (a) the system relaxes towards the same steady-state although with a *different power law* decay. In (b) the sudden quench of the interaction when $\eta_I = 0$ *alters the steady-current* j_S as well. This remains true for an arbitrary sequence of interaction quenches. We are able to write j_S as an explicit functional of the switching process and to establish that j_S is history dependent for non-smooth switchings.

The equilibrium Hamiltonian for the system of Fig. 1 reads

$$H_0 = H_R + H_L + \eta_I H_I + \eta_T H_T. \quad (1)$$

The one-body part of the left (L) and right (R) leads is $H_{R/L} = \mp i v_F \int_{-\infty}^{\infty} dx \psi_{R/L}^\dagger(x) \partial_x \psi_{R/L}(x)$, where the fermion field $\psi_{R/L}$ describes right/left moving electrons in lead R/L with Fermi velocity v_F (chiral leads). We take a density-density interaction of the form $H_I = \frac{1}{2} \int_{-\infty}^{\infty} dx [2g_2 \rho_R(x) \rho_L(x) + g_4(\rho_R^2(x) + \rho_L^2(x))]$, where $\rho_{R/L} \equiv : \psi_{R/L}^\dagger \psi_{R/L} :$ is (in standard notation) the fermionic density operator relative to the Fermi sea, and $g_{2/4}$ are the forward scattering couplings, corresponding to inter/intra lead interactions respectively. The two chiral liquids are linked at $x = 0$ via the tunneling term $H_T = \lambda \psi_R^\dagger(0) \psi_L(0) + \text{H.c.}$, which does not commute with

the total number of electrons $N_{R/L}$ of each lead.

If a bias $V = V_L - V_R$ is applied at, say, time $t = 0$, a finite current $j(t)$ starts flowing across the link. The current operator (in atomic units) $J = dN_L/dt = -dN_R/dt$ reads $J = i\lambda\psi_R^\dagger(0)\psi_L(0) + \text{H.c.}$. At zero temperature the current $j(t)$ is the TD average of J over the ground state $|\Psi_0\rangle$ of H_0 , i.e.,

$$j(t) = \langle \Psi_0 | J_{H_1}(t) | \Psi_0 \rangle, \quad (2)$$

where $J_{H_1}(t)$ is the J operator in the Heisenberg representation with respect to the interacting, contacted and biased Hamiltonian $H_1 = H_L + H_R + H_I + H_T + H_V$, $H_V = V_R N_R + V_L N_L$. Note that the factors η_I, η_T refer to times $t < 0$ and different values $\eta_I, \eta_T = 0, 1$ yields different H_0 and hence different initial states $|\Psi_0\rangle$. At positive times the Hamiltonian is the same in all cases.

The exact non-interacting solution. We start our analysis by calculating $j(t)$ when $\eta_T = 0$ (initially uncontacted) and $g_2 = g_4 = 0$ (always non-interacting). In terms of the Fourier transform $\psi_{k_{R/L}}$ of the original fermion fields, the current operator reads $J = (i\lambda/a) \sum_{kk'} \psi_{k_R}^\dagger \psi_{k'_L} + \text{H.c.}$, with a the usual short-distance cutoff. Its expectation value is then $j(t) = \lambda \text{Im} \sum_\alpha \int \frac{dp}{\pi} \Gamma_p^{R\alpha}(t) f_p^\alpha [\Gamma_p^{L\alpha}(t)]^*$ where the sum runs over $\alpha = R, L$, $f_p^{R/L} = f(\pm v_F p)$ is the Fermi function of lead R/L and $\Gamma_p^{\alpha\beta}(t) = -ia \int \frac{dk}{2\pi} \langle \Psi_0 | \psi_{k_\alpha} e^{-iH_1 t} \psi_{p_\beta}^\dagger | \Psi_0 \rangle$ is the sum of the probability amplitudes (retarded Green's functions) for the transition $p_\beta \rightarrow k_\alpha$. From the Dyson equation it is straightforward to find $\Gamma_p^{\alpha\alpha}(t) = -ie^{i(\alpha v_F p + V_\alpha)t}/(1 + c^2)$ and $\Gamma_p^{\bar{\alpha}\alpha}(t) = -ic \Gamma_p^{\alpha\alpha}(t)$, with $c = \lambda/(2v_F)$, and hence

$$j(t) = \frac{2c^2}{\pi(1 + c^2)^2} V. \quad (3)$$

The current is *discontinuous* in time; the steady-state value is reached instantaneously. This is due to the unbound (relativistic) energy spectrum[5] and the lack of interactions, as discussed in detail in Ref. 15. As we shall see, when $H_I \neq 0$ the transient regime is more complex.

Current to lowest order in λ . The problem does not have an exact solution when both H_I and H_T are present. Below, we calculate $j(t)$ to lowest order in λ . In general, perturbative treatments in the tunneling amplitude are a delicate issue[16]. In our case $j(t)$ has a Taylor expansion with convergence radius $\lambda < 2v_F$ for $H_I = 0$, see Eq. (3). We, therefore, expect a finite convergence radius at least for small interaction strengths. Let the unperturbed Hamiltonian be $\tilde{H}_0 = H_R + H_L + \eta_I H_I$ in equilibrium ($t < 0$) and $\tilde{H}_1 = H_R + H_L + H_I + H_V$ at positive times. At zero temperature and to lowest order in λ

$$j(t) = i \langle \tilde{\Psi}_0 | \int_0^t ds \left[H_{T, \tilde{H}_1}(s), J_{\tilde{H}_1}(t) \right] - \eta_T \int_0^{-i\infty} d\tau \left[H_{T, \tilde{H}_0}(\tau) J_{\tilde{H}_1}(t) + J_{\tilde{H}_1}(t) H_{T, \tilde{H}_0}(-\tau) \right] | \tilde{\Psi}_0 \rangle, \quad (4)$$

with $|\tilde{\Psi}_0\rangle$ the ground state of \tilde{H}_0 . The first term in the r.h.s. is the standard Kubo formula. Such term alone describes the transient response when the contacts are switched on at $t = 0$ ($\eta_T = 0$). If, however, the equilibrium system is already contacted ($\eta_T = 1$) we must account for a correction; this is the physical content of the second term[17]. At any finite time initial correlation effects are visible in both terms due to the ground state dependence on η_I . When $t \rightarrow \infty$ only the Kubo term survives, which yields the steady-current j_S . The dependence of j_S on the ground state ($\eta_I = 0, 1$) will be addressed below.

The averages in Eq. (4) can be explicitly calculated by resorting to the bosonization method[11]. We introduce the scalar fields ϕ and θ from $\rho_R(x) + \rho_L(x) = \frac{1}{\sqrt{\pi}} \partial_x \phi(x)$ and $\psi_{R/L}(x) = \frac{\kappa_{R/L}}{\sqrt{2\pi a}} e^{i\sqrt{\pi}[\phi(x) \mp \theta(x)]}$, with $\kappa_{R/L}$ the anticommuting Klein factors. The scalar fields obey the commutation relation $[\phi(x), \theta(x')] = i \text{sgn}(x - x')/2$. In terms of ϕ and θ the Hamiltonian $H = H_R + H_L + H_I$ is a simple quadratic form $H = \frac{v}{2} \int_{-\infty}^{\infty} dx [K^{-1}(\partial_x \phi(x))^2 + K \partial_x \theta(x)^2]$, with $v = \sqrt{(2\pi v_F + g_4)^2 - g_2^2}/2\pi$ the renormalized velocity and $K = \sqrt{(2\pi v_F + g_4 - g_2)/(2\pi v_F + g_4 + g_2)}$ a parameter which measures the interaction strength. Note that $0 < K \leq 1$ for repulsive interactions; $K = 1$ corresponds to the noninteracting case while small values of K indicate a strongly correlated regime.

By employing the gauge transformation[18] $\psi_{L,R} \rightarrow \psi_{L,R} e^{iV_{L/R}t}$ the problem of evaluating Eq. (4) is reduced to the calculation of different bosonic vacuum averages[11]. After some tedious algebra one finds

$$j(t) = \xi \text{Re} [\eta_T A_{\eta_I}(t) + B_{\eta_I}(t)], \quad (5)$$

where

$$A_0(t) = \sin(Vt) \int_0^\infty d\tau \gamma^2(t + i\tau),$$

$$B_0(t) = i \int_0^t ds \sin[V(s - t)] \gamma^{2K}(s - t) \times |\gamma(s - t)|^{(1-K)^2} \left| \frac{\gamma^2(s + t)}{\gamma(2t)\gamma(2s)} \right|^{1-K^2}, \quad (6)$$

for $\eta_I = 0$ and

$$A_1(t) = \sin(Vt) \int_0^\infty d\tau \gamma^{2K}(t + i\tau)$$

$$B_1(t) = i \int_0^t ds \sin[V(s - t)] \gamma^{2K}(s - t) \quad (7)$$

for $\eta_I = 1$, and where $\gamma(z) = a/(a - ivz)$ and $\xi = \lambda^2/(\pi a)^2$. In all cases ($\eta_I, \eta_T = 0, 1$) $j(t)$ is an odd function of V , as it should be. We also notice that for noninteracting systems ($K = 1$) we recover the expected result $A_1 = A_0$ and $B_1 = B_0$. In this case the function $\xi \text{Re}[B_{1,0}]$ coincides with the current in Eq. (3) to lowest

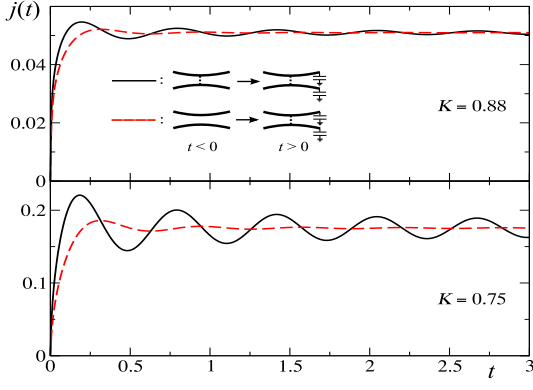


FIG. 2. Transient currents $j_{T1}(t)$ (solid) and $j_{T0}(t)$ (dashed) for $V = 10^{-2}$, $K = 0.88$ (upper panel), and $K = 0.75$ (lower panel). In the long-time limit they reach the same steady-state value. Current is in units of $\xi a/v$, V is in units of v/a and t is in units of $10^3 a/v$.

order in λ . We can now provide a quantitative analysis of the TD current response for different preparative configurations.

Contacted versus uncontacted ground state. We consider an initially contacted ($\eta_T = 1$) and uncontacted ($\eta_T = 0$) correlated ground state ($\eta_I = 1$) and compare the corresponding TD currents $j_{T1} \equiv \xi \text{Re}[A_1 + B_1]$ and $j_{T0} \equiv \xi \text{Re}[B_1]$. The current $j_{T0}(t)$ has been recently computed in Ref. [19]. In the long time limit it returns the well known steady-state result

$$j_S(\beta) = \sin(\pi K) \kappa(\beta) \text{sgn}(V) |V|^\beta \quad (8)$$

with $\kappa(\beta) = -\xi(a/v)^{\beta+1} \Gamma(-\beta) \sin(\beta\pi/2)$ and the exponent $\beta = 2K - 1$, obtained long ago by Kane and Fisher[20]. Since $A_1(t \rightarrow \infty) = 0$, j_{T1} approaches the same steady state. Note that the small bias limit is ill-defined for $K < 1/2$ due to the break down of the perturbative expansion in powers of λ [18, 21]. Even though $j_{T0}(t \rightarrow \infty) = j_{T1}(t \rightarrow \infty)$ the relaxation is different in the two cases, see Fig. 2. The function $j_{T0}(t)$ approaches the asymptotic limit with transient oscillations of frequency V and damping envelope proportional to t^{-2K} [19]. The more physical current j_{T1} , instead, decays much slower. The integral in $A_1(t)$ can be calculated analytically and yields

$$j_{T1}(t) - j_{T0}(t) = \xi a^{2K} \frac{\sin(Vt) \cos[(2K-1) \arctan(vt/a)]}{2v(2K-1)(a^2 + v^2 t^2)^{K-1/2}}, \quad (9)$$

which for long times decays as t^{1-2K} . (Equation (9) provides an independent, TD evidence that the perturbative treatment breaks down for $K < 1/2$.) Thus, an initially contacted state changes the power-law decay from $\sim t^{-2K}$ to the slower $\sim t^{1-2K}$. The amplitude of the transient oscillations is also significantly different, due to the factor $(2K-1)^{-1}$ in Eq. (9). For $K = 0.75$, j_{T1} oscillates with an amplitude about 10 times larger than that

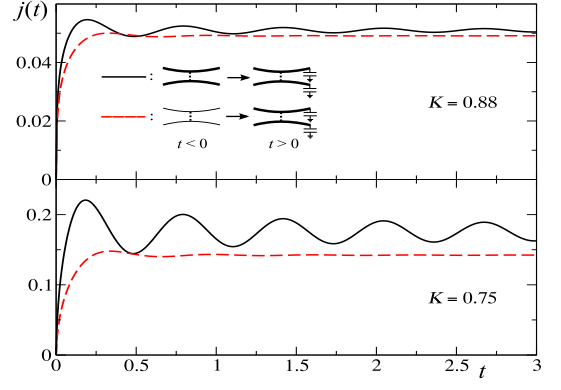


FIG. 3. Transient currents $j_{I1}(t)$ (solid) and $j_{I0}(t)$ (dashed) for $V = 10^{-2}$, $K = 0.75$ (upper panel) and $K = 0.88$ (lower panel). In the long-time limit they reach different steady-states. Same units as in Fig. (2).

of j_{T0} , see Fig. 2. The magnification of the oscillations was unexpected since for j_{T1} we only switch a bias while for j_{T0} also the contacts. This effect is not an artifact of the perturbative treatment: to support the validity of our results we checked that for $\eta_I = \eta_T = 1$ and zero bias the density matrix $\rho(t) = \langle \Psi_0 | \psi_{R,H1}^\dagger(t) \psi_{L,H1}(t) | \Psi_0 \rangle$ does not evolve in time to first order in λ (this is obvious for the exact density matrix). The constant value $\rho(t) = \rho(0)$ is the result of a subtle cancellation of TD functions similar to $A_1(t)$ and $B_1(t)$.

Correlated versus uncorrelated ground state. Next we consider the effects of correlations in the ground state. We take $\eta_T = 1$ and compare the TD currents j_{I1} and j_{I0} resulting from Eq. (5) when $\eta_I = 1$ and $\eta_I = 0$ respectively. Note that $j_{I1} \equiv j_{T1}$ (already calculated above). The current $j_{I0} = \xi \text{Re}[A_0 + B_0]$ is the response to a sudden bias switching and interaction quench; at $t > 0$ the electrons start tunneling from L to R and at the same time forming interacting quasiparticles. The interaction quench has a dramatic impact on the transport properties, both in the transient and steady-state regimes. From Fig. 3 we clearly see that the relaxation behavior is different. The damping envelope of $j_{I0}(t)$ is proportional to t^{-K^2-1} as opposed to t^{1-2K} of $j_{I1}(t)$. Notice that the exponent $-K^2 - 1 < 0$ for all K (first-order perturbation theory in λ is meaningful for all K).

In the long-time limit we find the intriguing result that $j_{I0}(t \rightarrow \infty)$ is exactly given by Eq. (8) with exponent $\beta = K^2$, thus suggesting that the structure of the formula (8) is *universal*. Below we will prove that this is indeed the case and that β is an elegant functional of the switching process. For now, we observe that *ground state correlations are not reproducible by quenching the interaction*. The system remembers them forever and steady-state quantities are inevitably affected. This behavior is reminiscent of the thermalization breakdown enlightened by Cazalilla[12] and others[13, 14]. Here, however,

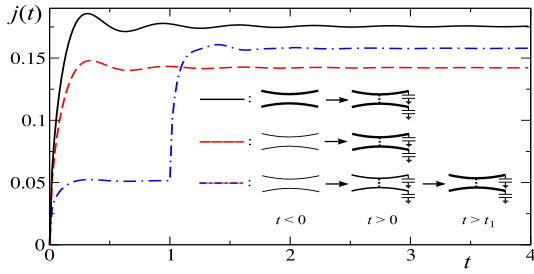


FIG. 4. Transient currents with $\eta_I = 1$, $\eta_T = 0$ (solid), and with $\eta_I = \eta_T = 0$ for the quench $1 \rightarrow K$ (dashed) and the quench sequence $1 \rightarrow \frac{1+K}{2} \rightarrow K$ (dotted-dashed). Here $K = 0.75$ and $V = 10^{-2}$ and the second quench occurs at $t_1 = 1$; same units as in Fig. (2).

we are neither in equilibrium nor close to it (the bias is treated to all orders). The non-equilibrium exponents $\beta = 2K - 1$ and $\beta = K^2$ refer to current-carrying states as obtained from the full TD Schrödinger equation with different initial states.

History dependence. We now address the question whether or not the physical steady-current $j_S(2K - 1)$ of Eq. (8) is reproducible by more sophisticated switching processes of the interaction like, e.g., an adiabatic switching. Preliminary insight can be gained by calculating $j(t)$ for a double quench: we first quench an interaction with $K_1 = (1 + K)/2$, let the system evolve, and then change suddenly $K_1 \rightarrow K_2 = K$. The current is calculated along the same line of reasoning of Eq. (4), although the formulas become considerably more cumbersome. In Fig. 4 we compare the TD currents for initially uncontacted leads resulting from an interaction K (solid), a single quench $1 \rightarrow K$ (dashed), and the aforementioned double quench (dotted-dashed). We clearly see that in the latter case the steady-current is larger than $j_S(K^2)$ (single-quench) and gets closer to $j_S(2K - 1)$. Strikingly, the double-quench steady-current is again given by $j_S(\beta)$ of Eq. (8) with $\beta = \frac{1}{2}(1 + K_1^2)(1 + (\frac{K_2}{K_1})^2) - 1$. This value of β depends only on the K -sequence and is independent of the quenching times. We have been able to extend the above solution to systems initially interacting with K_0 and then subject to an arbitrary sequence of quenches $K_0 \rightarrow K_1 \rightarrow \dots \rightarrow K_N = K$. We found the remarkable result that the formula (8) is *universal*, with the sequence dependent β given by

$$\beta[K_n] = \frac{K_0}{2^{N-1}} \prod_{n=0}^{N-1} \left[1 + \left(\frac{K_{n+1}}{K_n} \right)^2 \right] - 1. \quad (10)$$

This formula yields the correct values of β for the single and double quench discussed above. Note that for a sequence of increasing interactions $K_{n+1} \leq K_n$ it holds $\beta \geq 2K - 1$ with the equality valid only for $K_0 = K_1 = \dots = K_N = K$.

We now show that the special value $\beta = 2K - 1$ is also reproducible by an arbitrary (not necessarily adiabatic)

continuous ($N \rightarrow \infty$) sequential quenching. In this limit the variable $x_n = n/N$ becomes a continuous variable and we can think of the K_n as the values taken by a differentiable function $K(x)$ in $x = x_n$, with $K(0) = K_0$ and $K(1) = K$. Then, the quantity β becomes a functional of $K(x)$ that we now work out explicitly. Approximating $K(x_{n+1}) = K(x_n + \frac{1}{N}) \approx K(x_n) + \frac{1}{N} K'(x_n)$ and taking the logarithm of Eq. (10) we can write

$$\begin{aligned} \log \left(\frac{\beta[K(x)] + 1}{2K(0)} \right) &= \lim_{N \rightarrow \infty} \sum_{n=0}^{N-1} \log \left(1 + \frac{1}{N} \frac{K'(x_n)}{K(x_n)} \right) \\ &= \int_0^1 dx \frac{K'(x)}{K(x)} = \log \frac{K(1)}{K(0)}, \end{aligned} \quad (11)$$

from which it follows the history independent result

$$\beta[K(x)] = 2K - 1. \quad (12)$$

The above result can easily be generalized to discontinuous switching functions $K(x)$ for which, instead, the exponent β is history dependent.

Conclusions. In conclusion we studied the role of different preparative configurations in TD quantum transport between LLs. By using bosonization methods we showed that a sudden switching of the contacts does not change the steady-state but alters significantly the transient behavior, changing the damping envelope from $\sim t^{-2K}$ to $\sim t^{1-2K}$ and magnifying the amplitude of the oscillations. The effects of a sudden interaction quench is even more striking. Besides a different power law decay ($\sim t^{1-2K}$ versus $\sim t^{-K^2-1}$ damping envelope) the steady-current is also different; the I-V characteristic $j_S \propto V^\beta$ changes from $\beta = 2K - 1$ to $\beta = K^2$. More generally we proved that for a sequence of interaction quenches the steady-current is a universal function of the exponent β which, in turn, is a functional of the switching process. It is only for smooth switchings that β is history independent and equals the value $2K - 1$ of the initially interacting LL. The explicit β functional derived in this Letter establishes the existence of intriguing memory effects that point to a complex entanglement between equilibrium and non-equilibrium correlations in strongly confined systems.

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